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DIFFRACTION OF A PLANE HYDROACOUSTIC WAVE ON THE OPEN END OF A PLANE, SEMI-INFINITE WAVEGUIDE WITH THIN ELASTIC WALLS*

L. A. LEVITSKII

Acoustic steady state oscillations of a fluid with two parallel half-planes immersed in it and forming an open semi-infinite waveguide, are studied. The diffraction on the open end of a waveguide with perfectly rigid (soft) walls was studied in detail in /1/. The present paper deals with a waveguide with semi-transparent walls. The quantity sought is the pressure, for which the Helmholtz equation is assumed to hold inside the region, certain conditions containing high order derivatives at the boundary, and so-called boundary-contact conditions at the edge of the half-planes. The expressions for the boundary and boundary-contact operators are not given in concrete form. An exact expression for the pressure is obtained for the case when the acoustic field is generated by a plane wave.

1. Formulation of the problem. Two identical, thin semi-infinite parallel screens are placed in a compressible fluid to form an open, plane waveguide. We shall limit ourselves to the case of incompressible walls, since the compressibility of the material would lead to the problem of factorizing a second order matrix, and such problems have, in general, no analytic solution.

Let a plane hydroacoustic wave $A\exp[i(\mu x - \sqrt{k^2-\mu^2}y)]$ $(\mu = k\cos\varphi)$ impinge from the fluid on the waveguide at the angle φ to the screens (see Fig.1).

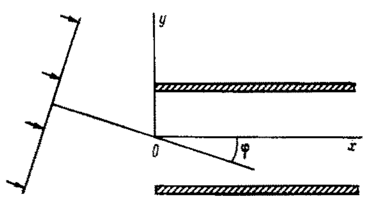

Fig.1

We seek to determine the field scattered from the walls of the waveguide. The factor $\exp(-i\omega t)$ determining the time dependence of the processes is neglected everywhere. We shall describe the acoustic processes in the system in terms of the pressure $P(x, y)$. The problem in question is that of constructing a solution of the homogeneous Helmholtz equation outside the walls of the waveguide ($k = w/c$ is the wave number, ω is the angular frequency, c is the speed of sound in the fluid and $2a$ is the width of the waveguide), with the boundary conditions at the walls

$$\Delta P(x, y) + k^2 P(x, y) = 0 \tag{1.1}$$

$$\frac{\partial}{\partial y} P(x, \pm a + 0) = \frac{\partial}{\partial y} P(x, \pm a - 0) \tag{1.2}$$

$$LP(x, y) = m_1\left(-\frac{\partial^2}{\partial x^2}\right)\frac{\partial}{\partial y} P(x, \pm a) + m_2\left(-\frac{\partial^2}{\partial x^2}\right)[P(x, \pm a + 0) - P(x, \pm a - 0)] = 0 \quad (x > 0) \tag{1.3}$$

Condition (1.2) describes the incompressibility of the walls and the continuity of the vertical displacements at the boundary between the fluid and the screen, and condition (1.3) follows from the equation of dynamics for plates /2/. The operators m_1 and m_2 are polynomials of the argument $\partial^2/\partial x^2$. The coefficients of these polynomials are functions of the mechanical parameters of the problem and can, generally speaking, depend on the wave number k. We assume, in accordance with the principle of limiting absorption, that $\operatorname{Im} k > 0$. A solution for $\operatorname{Im} k = 0$ is obtained by passing to the limit $\operatorname{Im} k \to 0$ /2/.

An important example of the condition of the form (1.3) is obtained by assuming that the walls of the waveguide are thin plates capable only of the flexural oscillations

$$\left(\frac{\partial^4}{\partial x^4} - \chi^4\right)\frac{\partial}{\partial y} P(x, \pm a) + \nu[P(x, \pm a + 0) - P(x, \pm a - 0)] = 0 \tag{1.4}$$

$$\chi = \chi_0\sqrt{k}, \quad \chi_0^4 = 12(1-\sigma^2)\rho_0 c^2 E^{-1}, \qquad \nu = \nu_0 k^2, \quad \nu_0 = 12(1-\sigma^2)\rho c^2 E^{-1}$$

Here σ is the Poisson's ratio, E is the Young's modulus, ρ_0 is the plate density and ρ denotes the fluid density. The quantities x, y, k and a are dimensionless, and the plate thickness h serves as the charateristic dimension. Another example of the boundary condition (1.3) is obtained by assuming that the walls of the waveguide are membranes.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\omega^2}{a^2}\right)\frac{\partial}{\partial y} P(x, \pm a) - \frac{\rho\omega^2}{a^2}[P(x, \pm a + 0) - P(x, \pm a - 0)] = 0, \qquad a^2 = \frac{T}{\rho_0}$$

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where T denotes the tension in the membrane and ρ_0 its density.

Since the conditions at the edges of the waveguide walls are not specified in the formulation (1.1)–(1.3), it follows that the solution of the problem is not unique. In order to obtain a unique solution, we must formulate the boundary-contact conditions determining the state at the wall edges (see Sect.4).

2. Construction of the solution. From the boundary conditions (1.2), (1.3), it is clear that the pressure $P(x, y)$ experiences a jump at the wall of the waveguide, while its normal derivative remains continuous. Let us write $P(x, y)$ as the potential of a double layer and introduce the pressure jumps at the walls: $f_1(x)$ at the wall $y = +a$ and $f_2(x)$ at the wall $y = -a$. Using the Green's function of the plane problem for the Helmholtz equation

$$G(\mathbf{r}, \mathbf{r}') = \pi i H_0^{(1)}(kr), \quad r = |\mathbf{r} - \mathbf{r}'|$$

where $H_0^{(1)}(kr)$ is the Hankel function, we have the following expression for the diffracted field:

$$P_p(x, y) = -\frac{1}{4\pi}\int_{-\infty}^{+\infty}\left[f_1(x')\frac{\partial}{\partial y}G(\mathbf{r}, \mathbf{r}_1') + f_2(x')\frac{\partial}{\partial y}G(\mathbf{r}, \mathbf{r}_2')\right]dx'$$

Here $\mathbf{r}_1'(x', a)$ and $\mathbf{r}_2'(x', -a)$ are the radius vectors of the points on the upper and the lower wall. Applying the theorem on Fourier transformation of a convolution, we obtain

$$P_p(x,y) = -\frac{i}{4\pi}\frac{\partial}{\partial y}\int_{-\infty}^{+\infty}\frac{\exp(i\lambda x)}{\sqrt{k^2-\lambda^2}}[F_1(\lambda)\exp(i\sqrt{k^2-\lambda^2}|y-a|+F_2(\lambda)\exp(i\sqrt{k^2-\lambda^2}|y+a|)]d\lambda$$

where $F_1(\lambda)$ and $F_2(\lambda)$ are Fourier transforms of the jumps $f_1(x)$ and $f_2(x)$. Now the principle of limiting absorption holds for $P_p(x, y)$, equation (1.1) is satisfied, $P(x, y)$ has a discontinuity at the walls and the derivative $P_y'(x, y)$ is continuous at $y = \pm a$. The scattered field is continuous near the edges of the waveguide walls. To ensure the continuity of $P(x, y)$ it is sufficient to demand that

$$F_1(\lambda) = O(\lambda^{-1-\varepsilon}), \quad F_2(\lambda) = O(\lambda^{-1-\varepsilon}), \quad 0 < \varepsilon < 1/2, \quad \lambda \to \pm\infty$$

Let us separate the complete field $P(x, y)$ into two parts: $P_s(x, y)$ symmetric in y, and $P_a(x, y)$ antisymmetric in y

$$P(x, y) = P_s(x, y) + P_a(x, y); \quad 2P_s(x, y) = P(x, y) + P(x, -y)$$

Below we shall study in detail the construction of a solution for the symmetric parts of the field. For the antisymmetric part we shall just quote the results, since the derivation is identical in both cases. We have

$$P_s(x, y) = A\exp(i\mu x)\cos(\sqrt{k^2-\mu^2}y) - \frac{i}{4\pi}\frac{\partial}{\partial y}\int_{-\infty}^{+\infty}\frac{\exp(i\lambda x)}{\sqrt{k^2-\lambda^2}}F_s(\lambda)[\exp(i\sqrt{k^2-\lambda^2}|y-a|) -$$

$$\exp(i\sqrt{k^2-\lambda^2}|y+a|)]d\lambda, \qquad 2F_s(\lambda) = F_1(\lambda) - F_2(\lambda)$$

Let us write the boundary condition (1.3) for the field $P_s(x, y)$

$$-A\exp(i\mu x)m_1(\mu^2)\sqrt{k^2-\mu^2}\sin(\sqrt{k^2-\mu^2}a) + \frac{1}{4\pi}\int_{-\infty}^{+\infty}F_s(\lambda)\exp(i\lambda x)l_s(\lambda)d\lambda = 0$$

$$l_s(\lambda) = i\sqrt{k^2-\lambda^2}m_1(\lambda^2)[1-\exp(2ai\sqrt{k^2-\lambda^2})] + 2m_2(\lambda^2)$$

Taking into account the fact that when $x > 0$ we have

$$\exp(i\mu x) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{\exp(i\lambda x)}{\lambda-\mu}d\lambda$$

we obtain the following integral equation:

$$\int_{-\infty}^{+\infty}\exp(i\lambda x)\left[F_s(\lambda)l_s(\lambda) + 2Aim_1(\mu^2)\sin(a\sqrt{k^2-\mu^2})\sqrt{k^2-\mu^2}\frac{1}{\lambda-\mu}\right]d\lambda = 0, \quad x > 0 \tag{2.1}$$

Since the jump in the acoustic pressure is equal to zero outside the plates, the second integral equation will have the form

$$\int_{-\infty}^{+\infty}\exp(i\lambda x)F_s(\lambda)d\lambda = 0 \quad (x < 0) \tag{2.2}$$

The roots of the dispersion equation

$$l_s(\lambda) = 0 \tag{2.3}$$

yield the wave numbers of the characteristic oscillations of the waveguide, symmetric in y.

We shall assume that the algebraic order $2S_1$ of the polynomial $m_1(\lambda^2)$ is not less than the degree $2S_2$ of the polynomial $m_2(\lambda^2)$, and the equation (2.3) with $\operatorname{Im} k > 0$ has no real roots on the basic sheet of the doubly-sheeted Riemann surface of the function $\sqrt{k^2-\lambda^2}$. We select the basic sheet of the radical $\sqrt{k^2-\lambda^2}$ in the following manner. We make a cut from the point $\lambda = k$ such, that it approaches asymptotically the line $\operatorname{Im}\sqrt{k^2-\lambda^2} = 0$ as $|\lambda| \to \infty$. We shall discuss the nature of the cut in more detail later. The second cut is made from the point $\lambda = -k$, and is symmetrical with respect to the first cut about the point $\lambda = 0$. We shall assume that on the basic sheet $\lim \operatorname{Im}\sqrt{k^2-\lambda^2} = +\infty$ as $\lambda \to \pm\infty$.

The roots of the equation (2.3) split into two groups. The roots of the first group (a finite number of roots) approach, as the walls of the waveguide move away from each other $(ka \to \infty)$, the roots of the dispersion equation

$$l(\lambda) = i\sqrt{k^2-\lambda^2}\, m_1(\lambda^2) + 2m_2(\lambda^2) = 0 \tag{2.4}$$

of the single elastic plate immersed in a fluid. The algebraic order of the function $l(\lambda)$ is equal to $2S_1+1$, therefore the equation (2.4) has $2(2S_1+1)$ roots on the Riemann surface. We shall call the corresponding roots of (2.3) the plate roots. The roots of the second group (their denumerable set) approach, as the density of the wall material increases $(\rho_0 \to \infty)$, the wave numbers $\zeta_N = \sqrt{k^2-(\pi N/a)^2}$ of the normal waves of a waveguide with perfectly rigid walls. The dispersion equation of this waveguide has the form

$$1 - \exp(2ai\sqrt{k^2-\lambda^2}) = 0$$

We shall call such roots the waveguide roots. We note that if the equation (2.3) is rewritten in the form

$$1 - \exp(2ai\sqrt{k^2-\lambda^2}) = \frac{2i}{\sqrt{k^2-\lambda^2}}\,\frac{m_2(\lambda^2)}{m_1(\lambda^2)} \tag{2.5}$$

then the right hand side of (2.5) will be of order of $1/\lambda^{(1+2S_1-2S_2)}$ as $|\lambda| \to \infty$. From this it follows that when $N \to \infty$, then the following asymptotics holds for the waveguide roots:

$$\zeta_N \sim i\sqrt{(\pi N/a)^2 - k^2}$$

and the roots approach the line

$$\operatorname{Im}\sqrt{k^2-\lambda^2} = 0$$

with increasing N

Let us now make a cut in the complex λ-plane from the point $\lambda = k$ in such a manner, that it passes through the waveguide roots of the equation (2.3) without interfering with the plate roots. Equations (2.1) and (2.2) will hold when the following conditions are fulfilled:

$$F_s(\lambda)\, l_s(\lambda) + \frac{2Ai}{\lambda-\mu}\, m_1(\mu^2)\sqrt{k^2-\mu^2}\sin(a\sqrt{k^2-\mu^2}) = u^+(\lambda), \qquad F_s(\lambda) = u^-(\lambda) \tag{2.6}$$

where $u^+(u^-)$ denotes a function analytic above (below) the contour of integration.

Eliminating $F_s(\lambda)$ from (2.6), we arrive at an inhomogeneous Riemann boundary value problem /3/ which consists of finding two functions $u^+(\lambda)$, $u^-(\lambda)$ from a linear relationship connecting these functions, which holds on the real axis.

Following the method of solving a similar Riemann problem given in /2/, we obtain

$$F_s(\lambda) = \frac{1}{l_s^-(\lambda)}\left[g_s(\lambda) + \frac{2Aim_1(\mu^2)}{(\mu-\lambda)\, l_s^+(\mu)}\sqrt{k^2-\mu^2}\sin(a\sqrt{k^2-\mu^2})\right] \tag{2.7}$$

$$l_s(\lambda) = l_s^+(\lambda)\, l_s^-(\lambda), \quad l_s^\pm(\lambda) = O(\lambda^{S_1+1/2}), \quad |\lambda| \to \infty$$

Here $g_s(\lambda)$ is a polynomial of degree $S_1 - 1$ with arbitrary coefficients to be determined later; $l_s^\pm(\lambda)$ is the result of factorizing the function $l_s(\lambda)$, and the function $l_s^+(\lambda)$ $(l_s^-(\lambda))$ is analytic in the upper (lower) half-plane.

Similarly, for the antisymmetric part of the field we have

$$F_a(\lambda) = \frac{1}{l_a^-(\lambda)}\left[g_a(\lambda) - \frac{2Am_1(\mu^2)}{(\mu-\lambda)\, l_a^+(\mu)}\sqrt{k^2-\mu^2}\cos(a\sqrt{k^2-\mu^2})\right]$$

$$l_a(\lambda) = i\sqrt{k^2-\lambda^2}\, m_1(\lambda^2)[1+\exp(2ai\sqrt{k^2-\lambda^2})] + 2m_2(\lambda^2)$$

where $l_a^\pm(\lambda)$ is the result of factorizing the function $l_a(\lambda)$.

3. Factorization of $l_s(\lambda)$.

We construct the factorization formula using the method given in /4/. According to Sect.2, function $l_s(\lambda)$ has no zeros on the strip $|\operatorname{Im}\lambda| < \tau$, $\tau < \operatorname{Im} k$. Clearly, a complex constant exists such that

$$\lim_{\operatorname{Re}\lambda\to\infty} l_s(\lambda)/(Q\gamma^{2S_1+1}) = 1, \quad |\operatorname{Im}\lambda| < \tau, \quad \gamma = \sqrt{\lambda^2-k^2} = -i\sqrt{k^2-\lambda^2}$$

As we said before, $l_s(\lambda)$ has a finite number of the plate roots lying outside the strip $|\operatorname{Im}\lambda| > \tau$ on the lower sheet of the Riemann surface, and an enumerable set of the waveguide roots lying on the cut. Let us construct the functions

$$F(\lambda) = \ln [l_s(\lambda) / (Q\gamma^{2S_1+1})]$$

For the difference

$$F(\lambda) - F(0) = \ln l_s(\lambda) - \ln l_s(0) - (S_1 + {}^1/_2) \ln (1 - \lambda^2 / k^2) \tag{3.1}$$

the Cauchy type integral is equal to

$$F(\lambda) - F(0) = \frac{1}{2\pi i} \int_{\Gamma_1+\Gamma_2} \frac{\lambda F(z)}{z(z-\lambda)} dz \tag{3.2}$$

The contour $\Gamma_1 + \Gamma_2$ is obtained by "expanding" a closed loop enclosing the points $z = 0$ and $z = \lambda$ (see Fig.2 where the light dots denote the waveguide roots of the equation (2.3), black dots denote the plate roots and dashed lines the cuts in the complex plane). The integrals over the infinitely distant parts of the contour parallel to the imaginary axis, are equal to zero. Using (3.1) and (3.2), we obtain the following expression for $l_s(\lambda)$:

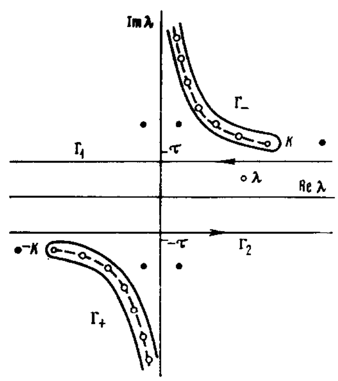

Fig.2

$$l_s(\lambda) = l_s(0)\left(1 - \frac{\lambda^2}{k^2}\right)^{(S_1+1/2)} \exp\left[\frac{1}{2\pi i} \int_{\Gamma_1+\Gamma_2} \frac{\lambda F(z)}{z(z-\lambda)} dz\right] \tag{3.3}$$

Every multiplying factor in (3.3) can be factorized simply, therefore we have

$$l_s^+(\lambda) = l_s^-(-\lambda) = \sqrt{l_s(0)}\,(1 + \lambda / k)^{S_1+1/2} \exp E_+(\lambda) \tag{3.4}$$

$$E_+(\lambda) = \frac{1}{2\pi i} \int_{\Gamma_2} \frac{\lambda F(z)}{z(z-\lambda)} dz \tag{3.5}$$

The function $E_+(\lambda)$ is bounded when $|\lambda| \to \infty$, therefore we have

$$l_s^+(\lambda) \sim \lambda^{S_1+1/2}, \quad |\lambda| \to \infty, \qquad \operatorname{Im} \lambda > -\tau$$

Integrating (3.5) by parts and replacing z by $-z$, we obtain

$$E_+(\lambda) = -\frac{1}{2\pi i} \int_{\Gamma_1} \left[\frac{2S_1+1}{k^2 - z^2} z + \frac{l_s'(z)}{l_s(z)}\right] \ln\left(1 + \frac{\lambda}{z}\right) dz$$

Let us now deform the contour Γ_1 into the curve Γ_- enclosing the cut, and take into account the residues of the function $l_s'(z) / l_s(z)$ at the plate zeros ξ_i of the function $l_s(z)$ lying in the upper half-plane of the lower sheet. This yields

$$l_s^+(\lambda) = \sqrt{l_s(0)}\,(1 + \lambda / k)^{S_1+1/2} \prod_{\operatorname{Im} \xi_i > 0} \left(1 + \frac{\lambda}{\xi_i}\right) \times \exp\left\{\frac{1}{2\pi i} \int_{\Gamma_-} \left[\frac{2S_1+1}{k^2 - z^2} z + \frac{l_s'(z)}{l_s(z)}\right] \ln\left(1 + \frac{\lambda}{z}\right) dz\right\} \tag{3.6}$$

We now transform the factorization formula (3.6) so as to separate the contribution of the waveguide poles of the function $l_s'(k) / l_s(z)$. Let us introduce a new variable $w = \sqrt{k^2 - z^2}$. Then $z = \sqrt{k^2 - w^2}$ where the branch of the square root is chosen so that $\operatorname{Im} \sqrt{k^2 - w^2} > 0$. The integral in (3.6) now becomes

$$Q_+(\lambda) = -\frac{1}{2\pi i} \int_{\Gamma_-} \left[\frac{2S_1+1}{w} - B_s(w)\right] \ln\left(1 + \frac{\lambda}{\sqrt{k^2 - w^2}}\right) dw, \qquad B_s(w) = \frac{1}{l_s(w)} \frac{dl_s(\sqrt{k^2 - w^2})}{dw}$$

Figure 3 depicts the contour Γ_- on the w-plane. The contour consists of two branches, Γ_-' and Γ_-''. Following /4/, we compute the integral along the small radius half-circle around the point $w = 0$. Denoting the residue of the function $B_s(w)$ at $w = 0$ by S_0, we find that the integral is equal to $(S_0 / 2 - S_1 - {}^1/_2) \ln (1 + \lambda / k)$. Performing the change $w \to -w$, we transform the integral along the branch Γ_-'' to an integral along the arc Γ_-''', and as a result we have

$$Q_+(\lambda) = (S_0 / 2 - S_1 - {}^1/_2) \ln (1 + \lambda / k) + \lim_{M \to \infty} \left[\sum_{n=1}^{\infty} \ln\left(1 + \frac{\lambda}{\zeta_n}\right) + \int_0^{\zeta_m} K_s(w) \ln\left(1 + \frac{\lambda}{\sqrt{k^2 - w^2}}\right) dw\right] \tag{3.7}$$

$$K_s(w) = -\frac{1}{2\pi i} [B_s(w) + B_s(-w)]$$

where the first term under the limit sign is a sum of residues of the function $B_s(w)$ in terms of the waveguide zeros ζ_n of the function $l_s(\lambda)$, and the second term is the integral along the cut up to the root ζ_m.

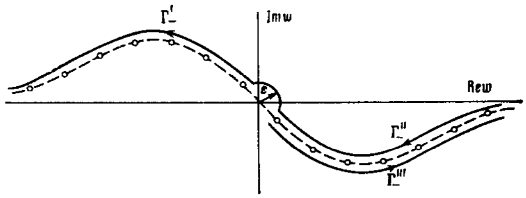

Fig.3

It can be shown that

$$K_s(w) = -a/\pi + K_{1s}(w) \tag{3.8}$$

$$K_{1s}(w) = -\frac{1}{\pi}\frac{\sin^2(aw)\,[\sqrt{k^2-w^2}m_1m_2 + w^2(m_1m_2' - m_1'm_2)] - a\sqrt{k^2-w^2}m_2^2}{\sqrt{k^2-w^2}\,[w^2\sin^2(aw)\,m_1^2 + w\sin(2aw)\,m_1m_2 + m_2^2]}$$

$$K_{1s}(w) = O(w^{-2(1+S_1-S_2)}),\ |w| \to \infty$$

In (3.8) the term $(-a/\pi)$ corresponding to a perfectly rigid waveguide, appears explicitly. We note that in (3.7) the integral of the function

$$-\frac{a}{\pi}\ln\left(1 + \frac{\lambda}{\sqrt{k^2-w^2}}\right)$$

along the cut, can be replaced by an integral along the segment, $w = 0$ to $w = M\pi/a$, of the real axis. Substituting the resulting limit, which has been computed in /4/, and the relations (3.8), (3.7) into (3.6), we finally obtain (C is the Euler constant)

$$l_s^+(\lambda) = l_s^-(-\lambda) = \sqrt{l_s(0)}\,(1+\lambda/k)^{S_0/2}\prod_{\mathrm{Im}\,\xi_i>0}\left(1+\frac{\lambda}{\xi_i}\right)\times \tag{3.9}$$

$$\prod_{n=1}^{\infty}\left(1+\frac{\lambda}{\zeta_n}\right)\exp\left(\frac{ia\lambda}{\pi n}\right)\exp\left[\frac{ia\lambda}{\pi}\left(1 - C + \ln\left(\frac{2\pi}{ka}\right) + i\frac{\pi}{2} + \right.\right.$$

$$\left.\left.\frac{ia\gamma}{\pi}\ln\left(\frac{\lambda-\gamma}{k}\right)\right]\exp\left[\int_0^\infty K_{1s}(w)\ln\left(1+\frac{\lambda}{\sqrt{k^2-w^2}}\right)dw\right]$$

The integral in (3.9) is taken along the cut. The formula for $l_a^+(\lambda)$ is obtained in the similar manner, with $K_{1s}(w)$ replaced by

$$K_{1a}(w) = -\frac{1}{\pi}\frac{\cos^2(aw)\,[\sqrt{k^2-w^2}m_1m_2 + w^2(m_1m_2' - m_1'm_2)] - a\sqrt{k^2-w^2}m_2^2}{\sqrt{k^2-w^2}\,[w^2\cos^2(aw)\,m_1^2 - w\sin(2aw)\,m_1m_2 + m_2^2]}$$

4. Boundary contact conditions.

We note that S_1 coefficients of the polynomial $g_s(\lambda)$ are, so far, arbitrary constants. To determine these constants, we must have S_1 boundary contact conditions specifying the state at the edges of the waveguide walls. The general form of the boundary contact conditions is the following:

$$R_jP(+0,a) = \lim_{x\to+0}\left\{S_{1j}\left(-i\frac{\partial}{\partial x}\right)\frac{\partial}{\partial y}P(x,a) + \right. \tag{4.1}$$

$$\left. S_{2j}\left(-i\frac{\partial}{\partial x}\right)[P(x,a+0) - P(x,a-0)]\right\} = 0,\quad j = 1, 2, \ldots, S_1$$

For a particular case of flexural oscillations (1.4) of a thin plate with a free edge we have $S_1 = 2$, and the boundary contact conditions have the form

$$\lim_{x\to+0}\frac{\partial^k}{\partial x^k}\frac{\partial}{\partial y}P(x,a) = 0,\quad k = 2, 3$$

In the physical sense, these conditions express the absence of concentrated forces and moments from the plate edges. The boundary contact conditions for the part of the field $P_s(x, y)$ symmetrical in y, and

$$R_jP_s(x,y) = -AS_{1j}(\mu)\sqrt{k^2-\mu^2}\sin(a\sqrt{k^2-\mu^2}) + \frac{1}{4\pi}\int_{-\infty}^{+\infty}\exp(+i0\lambda)F_s(\lambda)r_j(\lambda)\,d\lambda = 0,\quad j = 1, 2, \ldots, S_1 \tag{4.2}$$

where

$$r_j(\lambda) = iS_{1j}(\lambda)\sqrt{k^2-\lambda^2}\,[1-\exp(2ai\sqrt{k^2-\lambda^2})] + 2S_{2j}(\lambda),\qquad \int_{-\infty}^{+\infty}\exp(+i0\lambda)f(\lambda)\,d\lambda = \lim_{x\to+0}\int_{-\infty}^{+\infty}\exp(i\lambda x)f(\lambda)\,d\lambda$$

Writing explicitly the expansion of the polynomial $g_s(\lambda)$ in powers of λ

$$g_s(\lambda) = C_0 + C_1\lambda + \ldots + C_{s_1-1}\lambda^{s_1-1}$$

and substituting the expression for $F_s(\lambda)$ from (2.7) into (4.1), we obtain the following system of S_1 equations with S_1 unknown $C_n(k)$:

$$\sum_{n=0}^{S_1-1} C_n I_{jn} = 4\pi A S_{1j}(\mu)\sqrt{k^2-\mu^2}\sin(a\sqrt{k^2-\mu^2}) - \frac{2Am_1(\mu^2)\sqrt{k^2-\mu^2}\sin(a\sqrt{k^2-\mu^2})}{l_s^+(\mu)} J_j, \quad j = 1, 2, \ldots, S_1$$

$$I_{nj}(k) = \int_{-\infty}^{+\infty} \exp(+i0\lambda)\frac{l_s^+(\lambda)}{l_s(\lambda)}\lambda^n r_j(\lambda)\,d\lambda, \quad n = 0, 1, \ldots, S_1 - 1 \tag{4.3}$$

$$J_j(k) = \int_{-\infty}^{+\infty} \exp(+i0\lambda)\frac{l_s^+(\lambda)}{l_s(\lambda)}\frac{1}{\lambda-\mu} r_j(\lambda)\,d\lambda \tag{4.4}$$

The integrals in (4.3) and (4.4) are, in general, divergent. In order to regularize them /2/, we shall deform the contour of integration to a cut in the upper half-plane of λ. This will involve intersecting the poles of the integrand expression situated at the plate roots $l_s^-(\lambda)$

$$I_{nj}(k) = 2\pi i \sum_m \frac{\lambda^n r_j(\lambda)\, l_s^+(\lambda)}{l_s'(\lambda)}\Big|_{\lambda=\xi_m} + \int_{\Gamma_-} \frac{\exp(+i0\lambda)\lambda^n r_j(\lambda)}{l_s^-(\lambda)}\,d\lambda \tag{4.5}$$

The summation in (4.5) is carried out over the plate roots ξ_m of the equation (2.3), lying in the upper half-plane of λ. Taking into account the by-pass relations for the functions $l_s^-(\lambda)$ obtained in /2/, we have

$$l_s^-(\lambda)\Big|_{\lambda\in\Gamma_-''} = \frac{l_s^\circ(\lambda)}{l_s(\lambda)} l_s^-(\lambda)\Big|_{\lambda\in\Gamma_-'}, \qquad l_s^\circ(\lambda) = -\sqrt{k^2-\lambda^2}m_1(\lambda^2)\left[1 - \exp(-2ai\sqrt{k^2-\lambda^2})\right] + 2m_2(\lambda^2)$$

The values of the functions $l_s^\circ(\lambda)$ on the basic sheet coincide with the values of $l_s(\lambda)$ on the second sheet.

Now we can replace the integral along Γ_- by an integral along the edge Γ_-''

$$I_{nj}(k) = 2\pi i \sum_m \frac{\lambda^n r_j(\lambda)\, l_s^+(\lambda)}{l_s'(\lambda)}\Big|_{\lambda=\xi_m} + 8i\int_{\Gamma_-''} \exp(+i0\lambda)\frac{\lambda^n\sqrt{k^2-\lambda^2}\sin^2(a\sqrt{k^2-\lambda^2})}{l_s^-(\lambda)\, l_s^\circ(\lambda)}\left[m_2S_{1j} - m_1S_{2j}\right]d\lambda \tag{4.6}$$

To make the integral in (4.6) convergent, we impose the following restriction on the function $r_j(\lambda)$:

$$\left[m_2(\lambda^2)\, S_{1j}(\lambda) - m_1(\lambda^2)\, S_{2j}(\lambda)\right] = o(\lambda^{2S_1}), \quad |\lambda| \to \infty \tag{4.7}$$

i.e. the function appearing in the left hand part of (4.7) increases at infinity more slowly than $|\lambda|^{2S_1}$. As was said in /2/, a restriction of the type (4.7) imposed on $r_j(\lambda)$ implies the necessity of existence of a relationship connecting the boundary contact operators R_j with the boundary operator L, as a direct consequence of the physical features of the phenomenon. Similarly, for the integrals J_j we obtain the expression

$$J_j(k) = 2\pi i \sum_m \frac{r_j(\lambda)\, l_s^+(\lambda)}{(\lambda-\mu)\, l_s'(\lambda)}\Big|_{\lambda=\xi_m} + 2\pi i\frac{r_j(\mu)}{l_s^-(\mu)} + 8i\int_{\Gamma_-''} \frac{\sqrt{k^2-\lambda^2}\sin^2(a\sqrt{k^2-\lambda^2})\left[m_2S_{1j} - m_1S_{2j}\right]}{l_s^-(\lambda)\, l_s^\circ(\lambda)}\,d\lambda$$

Expressions for the boundary contact integrals for the part of the field $P_a(x, y)$ antisymmetric with respect to y, are obtained in the same manner.

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